

Spin-orbital motion and Thomas precession in the classical and quantum theories

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Abstract

The motion of a magnetic spin particle in electromagnetic fields is considered on the basis of general principles of the classical relativistic theory. Alternative approaches in derivation of the equations of charge motion and spin precession, the problem of noncollinearity of the momentum and velocity of a particle with spin, the origin and the meaning of Thomas precession in dynamics of the spin particle are also considered. The correspondence principle in the spin theory is discussed.

1 Introduction

In the initial form the classical spin theory was constructed by Ya.I. Frenkel [1] almost at the same time when the spin was discovered. The main theses of quantum spin theory were formulated by W. Pauli. Later with appearance of the Dirac relativistic equation the quantum spin theory was consecutively developed.

For a long time the classical and quantum spin theories developed independently from each other. It seemed that there was no connection between them. A large number of authors, including the authors of well-known books [2, 3, 4], assumed that there was no the classical spin theory in general (see also [5]).

Later the Bargmann-Michel-Telegdi (BMT) classical spin equation [6] in accordance with the Frenkel spin equation (see [7]) was obtained on the basis of the relativistic quasiclassical spin theory [8, 9-11] and the Heisenberg equations of operators motion in the relativistic quantum mechanics (see [12]). After that the situation has changed. BMT equation successfully confirmed the results of experiments on precision determination of the anomalous magnetic moment of an electron [13, 14], muon [15] and also the masses of a number of heavy elementary particles [16]. As it turned out well to explain the experimentally observed effect of radiative self-polarization of electrons [17, 18] within the frameworks of the quasiclassical spin theory as well as the Dirac quantum theory. Moreover, it was found that even purely quantum effect such as spin-flip radiation has classical description [19].

Nevertheless there is no united point of view about the fundamental principles of construction of the classical spin theory in the modern scientific and educational literature. There is a great

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variety of the classical methods of spin description, which are not confirmed with each other in many respects (see, for example, [20]). In this work the attempt in constructing of a correct classical spin theory was made on the basis of the general principles of classical electrodynamics. Of course, the classical and the quantum spin theories have to be correlated at $\hbar \rightarrow 0$.

A discussion of this problem is actual because many problems of the spin properties of elementary particles (noncollinearity of the particle momentum and velocity, the origin of the Thomas precession and emission of spin particles etc.) can be apparently resolved only by means of the classical theory. The correctly formulated classical spin theory allows also to understand more profound some of purely quantum properties of spin particles as phenomena of creation of pairs and Zitterbewegung, the problem of maximal acceleration et al.

2 Equations of motion of a spin particle in the classical electrodynamics

Here we obtain the equations of motion of a classical spin particle by means of the Lagrange method and the energy-momentum tensor formalism.

2.1 Lagrangian formulation of the problem

We introduce the Lagrange function of a charged particle with intrinsic magnetic moment in the additive form

$$L = L_e + L_s,$$

where

$$L_e = \frac{1}{2} m_0 v_\rho v^\rho + \frac{e}{c} v_\rho A^\rho$$

is the well-known Lagrange function of the relativistic charged particle (see, for example, [21]),

$$L_s = \frac{\mu}{2} H_{\alpha\beta} \Pi^{\alpha\beta}$$

is the spin addition taking into consideration the interaction between the intrinsic magnetic moment of the particle and an external electromagnetic field.

In the adduced above relations the following notations were taken: $v^\rho = (v^0, \mathbf{v}) = c\gamma(1, \boldsymbol{\beta})$ is the four-dimensional velocity of a particle, $\gamma = 1/\sqrt{1 - \beta^2}$ is the Lorentz-factor, $A^\rho = (\varphi, \mathbf{A})$ are the potentials of an external field, $H^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the electromagnetic field tensor. We take the metric $g^{\mu\nu} = (-1, 1, 1, 1)$ and therefore $v_\rho v^\rho = -c^2$, $H^{10} = -E_x$, $H^{12} = H_z$ etc. Spin tensor $\Pi^{\alpha\beta}$ is separated out the tensor of the intrinsic magnetic moments $\mu^{\alpha\beta}$ (the Frenkel tensor) and therefore it is dimensionless

$$\mu^{\alpha\beta} = \mu \Pi^{\alpha\beta}, \quad \mu = g\mu_0 s.$$

Here $\mu_0 = e\hbar/2m_0c$ is the Bohr magneton, s is the spin number, which we put equal to $1/2$. It is supposed that g -factor contains the anomalous part. As known $g = 2(1 + \alpha/2\pi)$ for an electron.

The tensor $\Pi^{\alpha\beta}$ is space-like and satisfies the Frenkel condition [1]

$$v_\alpha \Pi^{\alpha\beta} = 0. \tag{1}$$

The more acceptable functional dependence in L is given by the formally equivalent expression

$$L = \frac{1}{2} m_0 \left(1 - \frac{\mu}{2m_0 c^2} H_{\alpha\beta} \Pi^{\alpha\beta} \right) v_\rho v^\rho + \frac{e}{c} v_\rho A^\rho + \frac{\mu}{4} H_{\alpha\beta} \Pi^{\alpha\beta}. \quad (2)$$

The first term contains a factor

$$m = m_0 \left(1 - \frac{\mu}{2m_0 c^2} H_{\alpha\beta} \Pi^{\alpha\beta} \right), \quad (3)$$

which can be presented as the mass of a particle with spin. The spin mass (3) was introduced in the works of many authors (see [7, 20, 22-31] et al.) by various means.

Substituting (2) into the Euler-Lagrange relativistic equations we obtain

$$\frac{d}{d\tau} (m v^\rho) = \frac{e}{c} H^{\rho\lambda} v_\lambda + \frac{\mu}{2} \Pi_{\alpha\beta} \partial^\rho H^{\alpha\beta}, \quad (4)$$

where τ is the proper time. The equation (4) can be also presented in the form

$$m \frac{dv^\rho}{d\tau} = \frac{e}{c} H^{\rho\lambda} v_\lambda + \frac{\mu}{2} \Pi_{\alpha\beta} \partial^\rho H^{\alpha\beta} - \frac{dm}{d\tau} v^\rho. \quad (5)$$

The calculation of $dm/d\tau$ can be carried out only after formulation of spin equations of motion (see further) and then we can obtain with an accuracy to terms of orders \hbar^2 , $\hbar(g-2)$

$$\frac{dm}{d\tau} = -\frac{\mu}{2c^2} v_\lambda \Pi_{\alpha\beta} \partial^\lambda H^{\alpha\beta}.$$

In this case equation (5) really takes the space-like form (see [7])

$$m \frac{dv^\rho}{d\tau} = \frac{e}{c} H^{\rho\lambda} v_\lambda + \frac{\mu}{2} \Pi_{\alpha\beta} \vec{\partial}^\rho H^{\alpha\beta}, \quad (6)$$

where

$$\vec{\partial}^\rho = \partial^\rho + \frac{1}{c^2} v^\rho v_\lambda \partial^\lambda$$

is the space-like derivative.

The equation (4) was obtained by Frenkel [1] in the other form by means of the Lagrange variational multipliers.

2.2 Energy-momentum tensor formalism

The density of the energy-momentum tensor of a spin particle is represented as (radiation field is neglected)

$$\mathcal{P}^{\alpha\beta} = \mathcal{P}_e^{\alpha\beta} + \tilde{\mathcal{P}}^{\alpha\beta},$$

where

$$\mathcal{P}_e^{\alpha\beta} = c \int_{-\infty}^{\infty} d\tau' m v^\alpha(\tau') v^\beta(\tau') \delta(R^\rho - r^\rho(\tau')) \quad (7)$$

is the energy-momentum tensor density of the charge, τ' is the invariant parameter which after integration with δ -function

$$\delta(cT - ct(\tau')) = \frac{\delta(\tau - \tau')}{c\left(\frac{dt}{d\tau'}\right)_{T=t}} = \frac{1}{c\gamma}\delta(\tau - \tau')$$

gives the proper time τ :

$$\mathcal{P}_e^{\alpha\beta} = mv^\alpha(\tau)u^\beta(\tau)\delta(\mathbf{R} - \mathbf{r}(\tau)), \quad (8)$$

where $u^\beta(\tau) = (c, \mathbf{u})$, \mathbf{u} is ordinary velocity of the particle.

The formula (7) is written under assumption that a force acting on the particle is defined by the relation $F^\alpha = mw^\alpha$. In the general case $F^\alpha = dP^\alpha/d\tau$, where P^α is the momentum of the particle. Then instead of (7) and (8) we have

$$\mathcal{P}_e^{\alpha\beta} = c \int_{-\infty}^{\infty} d\tau' P^\alpha(\tau') v^\beta(\tau') \delta(R^\rho - r^\rho(\tau')) \quad (9)$$

and

$$\mathcal{P}_e^{\alpha\beta} = P^\alpha(\tau) u^\beta \delta(R^\rho - r^\rho). \quad (10)$$

The energy-momentum tensor of electromagnetic field is specified, in the usual fashion, by expression

$$\tilde{\mathcal{P}}^{\alpha\beta} = -\frac{1}{4\pi} \left(H^{\alpha\rho} H_\rho^\beta + \frac{1}{4} g^{\alpha\beta} H_{\rho\gamma} H^{\rho\gamma} \right).$$

In the differential form the conservation energy-momentum law has the form

$$D_\beta \mathcal{P}^{\alpha\beta} = 0,$$

where $D_\beta = \partial/\partial R^\beta$. Using the ordinary manner (see, e.g., [32]) one can integrate the expression over the four-volume $d\Upsilon$. Then omitting details of calculations ¹ we obtain

$$\frac{d}{d\tau} mv^\alpha = \frac{e}{c} H^{\alpha\beta} v_\beta + \frac{\mu}{2} \Pi_{\rho\lambda} \partial^\alpha H^{\rho\lambda} \quad (11)$$

in the case (8) and

$$\frac{dP^\alpha}{d\tau} = \frac{e}{c} H^{\alpha\beta} v_\beta + \frac{\mu}{2} \Pi_{\rho\lambda} \partial^\alpha H^{\rho\lambda} \quad (12)$$

in the case (10).

It is easy to see that the first equation (11) coincides with the equation obtained by the Lagrangian method. The second equation (12), generally speaking, differs from the above equation because the momentum P^α is not determined. In the following we shall use the more general second case. As we shall see later, the problem of agreement between equations (11) and (12) is not trivial and will be discussed below.

¹Here we take into account that total current density is made up the current density of the charge j_e^α and the current density of the magnetic moment j_s^α , that is

$$j^\alpha = j_e^\alpha + j_s^\alpha,$$

where

$$j_e^\alpha = ec \int d\tau' v^\alpha(\tau') \delta(R^\rho - r^\rho(\tau')), \quad j_s^\alpha = \mu c^2 \int d\tau' \Pi^{\alpha\lambda}(\tau') D_\lambda \delta(R^\rho - r^\rho(\tau')).$$

3 Spin equations and the problem of noncollinearity of momentum and velocity

Let us start by considering the successive derivation of the spin equations in the classical relativistic mechanics.

3.1 Formalism of angular momentum tensor

We introduce the density tensor of total angular momentum of a particle in the electromagnetic field

$$M^{\alpha\lambda\beta} = R^{[\alpha}\mathcal{P}^{\lambda\beta]} = R^\alpha\mathcal{P}^{\lambda\beta} - R^\beta\mathcal{P}^{\lambda\alpha}.$$

The conservation law of angular momentum in differential form is

$$D_\lambda M^{\alpha\lambda\beta} = 0.$$

If we then act as in section 2.2. one obtains the following equation of motion of the angular momentum $M^{\alpha\beta}$

$$\frac{dM^{\alpha\beta}}{d\tau} = \mu H^{[\alpha\rho}\Pi_\rho^{\beta]} + r^{[\alpha} \left(\frac{e}{c} H^{\beta]\rho} v_\rho + \frac{\mu}{2} H_{\rho\lambda} \partial^{\beta]} \Pi^{\rho\lambda} \right),$$

or, taking into account the momentum form of the equation of motion (12),

$$\frac{dM^{\alpha\beta}}{d\tau} = \frac{d}{d\tau} \left(r^{[\alpha} P^{\beta]} \right) - v^{[\alpha} P^{\beta]} + \mu H^{[\alpha\rho}\Pi_\rho^{\beta]}. \quad (13)$$

Using the definition of orbital momentum

$$L^{\alpha\beta} = r^\alpha P^\beta - r^\beta P^\alpha$$

we can represent the equation (13) as

$$\frac{dM^{\alpha\beta}}{d\tau} = \frac{d}{d\tau} \left(L^{\alpha\beta} + S^{\alpha\beta} \right).$$

It follows that $S^{\alpha\beta}$ is the spin moment of the particle. By separating from $S^{\alpha\beta}$ the factor $\hbar/2$ we come to the dimensionless spin equation

$$\frac{\hbar}{2} \frac{d\Pi^{\alpha\beta}}{d\tau} = -v^{[\alpha} P^{\beta]} + \mu H^{[\alpha\rho}\Pi_\rho^{\beta]}. \quad (14)$$

It is also evident that $\Pi_{\alpha\beta} \left(d\Pi^{\alpha\beta}/d\tau \right) = 0$, i.e. $\Pi_{\alpha\beta}\Pi^{\alpha\beta} = \text{const}$ is well known spin invariant. We emphasize that equation (14) was derived under assumption that the Frenkel condition (1) is valid.

3.2 Spin equations in the Frenkel formalism

If, as usual, the momentum of a particle is defined by relation $P^\alpha = mv^\alpha$, i.e. momentum and velocity are collinear, the spin equation (14) can be simplified to

$$\frac{\hbar}{2} \frac{d\Pi^{\alpha\beta}}{d\tau} = \mu H^{[\alpha\rho} \Pi_{\rho}^{\beta]}.$$

This equation differs essentially from experimentally verified BMT equation since formally it does not contain in the pure state the terms with anomalous magnetic moment $\mu_a = \mu - \mu_0 = \mu_0(g - 2)/2$. Matters can be improved by replacement

$$P^\alpha = mv^\alpha + \vec{Z}^\alpha. \quad (15)$$

Additional term \vec{Z}^α must be concerned with anomalous magnetic moment in a way. In general, it leads to noncollinearity of the momentum and the velocity of the spin particle. Taken along, this fact is not unexpected. It is reasonable that the spin particle moving along curved trajectory executes a peculiar trembling motion (Zitterbewegung).

Notice that the relation of the type (15) was first adduced in the original Frenkel theory [1]. Thus we can say that the first definition of spin mass m was made by Ya.I.Frenkel (m is the Langrange uncertain multiplier λ in the Frenkel theory).

Next we shall assume that the additional term \vec{Z}^α is space-like vector which satisfies condition

$$v_\alpha \vec{Z}^\alpha = 0. \quad (16)$$

It follows that in the system of rest the particle energy is equal to mc^2 and contains the potential energy of the intrinsic magnetic moment of the particle (see definition m in (3)). According to (15) and (16) we obtain

$$v_\alpha P^\alpha = -mc^2. \quad (17)$$

By substituting (15) into the spin equation (14) we have

$$\frac{\hbar}{2} \frac{d\Pi^{\alpha\beta}}{d\tau} = -v^{[\alpha} \vec{Z}^{\beta]} + \mu H^{[\alpha\rho} \Pi_{\rho}^{\beta]}. \quad (18)$$

It is interesting that the spin mass drops out of the equation. To determine \vec{Z}^α equation (18) should be multiplied by v_β . Then we find

$$\vec{Z}^\alpha = \frac{\hbar}{2c^2} \Pi^{\alpha\beta} w_\beta - \frac{\mu}{c^2} \Pi^\alpha{}_\rho H^{\rho\beta} v_\beta. \quad (19)$$

Rewriting \vec{Z}^α with regard for the equation of motion (6) and the Frenkel condition (1) we obtain

$$\vec{Z}^\alpha = \Pi^{\alpha\beta} \left(\frac{\mu\hbar}{4mc^2} \Pi_{\rho\lambda} \vec{\partial}_\beta H^{\rho\lambda} - \frac{\mu_a}{c^2} H_{\beta\rho} v^\rho \right). \quad (20)$$

Thus, here we can see not only the expected anomalous magnetic moment μ_a but also a gradient in field term which is proportional to \hbar . Since $\vec{Z}^\alpha \sim \hbar^2$, $\hbar(g - 2)$ the spin mass in (20) is replaced by the rest mass m_0 .

It is significant from definition of \vec{Z}^α (20) it follows that a free particle does not undergo the spin perturbation of trajectory. If we assume that $g = 2$ (anomalous magnetic moment is absent) \vec{Z}^α will coincide with the expression found by Frenkel² in a different way

$$\vec{Z}^\alpha|_{g=2} = \frac{1}{4} \frac{\mu_0 \hbar}{m_0 c^2} \Pi^{\alpha\beta} \Pi_{\rho\lambda} \vec{\partial}_\beta H^{\rho\lambda}.$$

For arbitrary values of g the expressions \vec{Z}^α in (19) and (20) are associated with definition of the momentum in the Corben theory [24].

By substituting \vec{Z}^α (20) into equation (18) we obtain the spin equation

$$\frac{d\Pi^{\alpha\beta}}{d\tau} = \frac{eg}{2m_0 c} H^{[\alpha\rho} \Pi_{\rho}^{\beta]} + v^{[\alpha} \Pi^{\beta]\rho} \left(\frac{g-2}{2} \frac{e}{m_0 c^3} H_{\rho\lambda} v^\lambda - \frac{\mu}{2m_0 c^2} \Pi_{\eta\lambda} \vec{\partial}_\rho H^{\eta\lambda} \right). \quad (21)$$

This equation is written with an accuracy to terms of orders \hbar , $g-2$. The spin equation (21) was first obtained by H. Corben [24] (see also [21, 34]). In particular case of uniform fields this equation transforms to the tensor form of the BMT spin equation (see [7, 35]).

Now we can refine the expression $dm/d\tau$ in the equation (5). According to (17) and (12) we have³

$$\frac{dm}{d\tau} = -\frac{1}{c^2} \left(w_\alpha P^\alpha + v_\alpha \frac{dP^\alpha}{d\tau} \right) \approx -\frac{\mu}{2c^2} \Pi_{\rho\lambda} v_\alpha \partial^\alpha H^{\rho\lambda}.$$

This expression is written with an accuracy to \hbar^2 , $\hbar(g-2)$ since we assume that the equation of charge motion has an accuracy of the first order in \hbar .

Hence, if we substitute $dm/d\tau$ in the equation of charge motion (5) it will taken the form of the equation (6) considered in subsection 2.1.

3.3 Spin equation in the Shirokov formalism

Instead of the Frenkel condition (1) many authors (see, for example, in Ref. [20, 37]) use the auxiliary condition introduced by Yu.M. Shirokov [38]⁴

$$P_\alpha \Pi^{\alpha\beta} = 0. \quad (22)$$

²In notation of Frenkel we have (see formula (22a) in Ref. [1])

$$\vec{Z}^\alpha = -\mu^{\alpha\beta} a_\beta, \quad a^\beta = -\frac{1}{2e_0 c} \mu_{\rho\lambda} \partial^\beta H^{\rho\lambda}.$$

³The exact expression $dm/d\tau$ has the form (see also [36])

$$\frac{dm}{d\tau} = \frac{\mu}{c^4} w_\alpha \Pi^{\alpha\rho} H_{\rho\beta} v^\beta - \frac{\mu}{2c^2} \Pi_{\rho\lambda} v_\alpha \partial^\alpha H^{\rho\lambda}.$$

However, it can be shown that the first term has just an accuracy of \hbar^2 , $\hbar(g-2)$.

⁴It will be noted that the condition (3) was first formulated by Yu.M. Shirokov for spin four-vector. As it is known (see, for example, [33]) the spin four-vector S^μ is connected with tensor $\Pi^{\alpha\beta}$ by relation

$$S^\mu = \frac{1}{2m_0 c} \varepsilon^{\mu\nu\alpha\beta} \Pi_{\alpha\beta} P_\nu, \quad \Pi^{\alpha\beta} = \frac{1}{m_0 c} \varepsilon^{\alpha\beta\rho\sigma} S_\rho P_\sigma.$$

It follows that in the Shirokov formalism condition (3) have the form $P_\mu S^\mu = 0$. Sometimes the condition (3) is called as Nakano condition [39] (see also [40]).

In this case the intrinsic angular momentum is specified with respect to the inertial center of the spin particle in the covariant form. The classical Zitterbewegung is a result of the fact that coordinates of the inertial center do not coincide with coordinates of the particle.

On the lines of Shirokov formalism the expansion of the momentum (15) can be rewritten in the form

$$P^\alpha = mv^\alpha + Z^\alpha, \quad (23)$$

where vector Z^α satisfies the condition [41]

$$P_\alpha Z^\alpha = 0. \quad (24)$$

By analogy with preceding case we obtain instead of (19) the following expression

$$Z^\alpha = \frac{m}{P_\alpha P^\alpha} \left(-\frac{\hbar}{2} \Pi^{\alpha\beta} \frac{dP_\beta}{d\tau} + \mu \Pi^\alpha{}_\rho H^{\rho\beta} v_\beta \right). \quad (25)$$

With due regard for smallness of square $Z_\alpha Z^\alpha \sim \hbar^2$ we have

$$P_\alpha P^\alpha \approx -m^2 c^2. \quad (26)$$

It should be noted that this relation remains valid also within the framework of Frenkel formalism.

Using the equation of motion (12), conditions (24) and relations(26) we come to the following expression

$$Z^\alpha = \Pi^{\alpha\beta} \left(\frac{\mu \hbar}{4m_0 c^2} \Pi_{\rho\lambda} \partial_\beta H^{\rho\lambda} - \frac{\mu_a}{c^2} H_{\beta\rho} v^\rho \right). \quad (27)$$

for the linear approximation in field.

The expression (27) differs from \vec{Z}^α in (20) only by derivative (there is ∂^α instead of $\vec{\partial}^\alpha$). However, it is unimportant what derivative presents in the spin equations in the Frenkel formalism, $\vec{\partial}^\alpha$ and ∂^α , as

$$\Pi_{\alpha\beta} \partial^\beta = \Pi_{\alpha\beta} \vec{\partial}^\beta.$$

It is very interesting that in the Shirokov formalism the spin equation (14) has the same form as in the Frenkel formalism. Therefore, in the Shirokov and Frenkel formulations the spin equations coincide with the equation (21) up to terms linear in the external field. As far as the charge equation of motion are concerned they coincide with equations (11) and (12) in the linear approximations in powers of \hbar and in field (see also in Ref. [36], p.51).

4 Thomas precession as a kinematical spin-orbit effect

It is best to perform the analysis of the spin equations of motion by use of spin vector ζ given in the rest frame. This vector is connected with components of the tensor $\Pi^{\alpha\beta}$ by means of Lorentz transformation

$$\zeta = \frac{1}{\gamma} \Pi + \frac{\gamma}{\gamma + 1} \beta (\beta \Pi)$$

and the corresponding spin equation has the form

$$\begin{aligned} \frac{d\boldsymbol{\zeta}}{d\tau} = & \frac{eg}{2m_0c}[\boldsymbol{\zeta}\mathbf{H}] - \frac{e(g-2)}{2m_0c} \frac{\gamma^2}{\gamma+1}[\boldsymbol{\zeta}[\boldsymbol{\beta}\mathbf{H}]] - \frac{e}{m_0c}\gamma \left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) [\boldsymbol{\zeta}[\boldsymbol{\beta}\mathbf{E}]] + \\ & + \frac{ge\hbar}{4m_0^2c^2} \frac{\gamma^2}{\gamma+1}[\boldsymbol{\zeta}[\boldsymbol{\beta}\boldsymbol{\nabla}]] \left\{ (\boldsymbol{\zeta}\mathbf{H}) + (\boldsymbol{\beta}[\boldsymbol{\zeta}\mathbf{E}]) - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\boldsymbol{\zeta})(\boldsymbol{\beta}\mathbf{H}) \right\}. \end{aligned} \quad (28)$$

This equation is known in literature (see, for instance, [42]) for particular case of homogeneous fields. In this case the equation of spin precession can be presented in the form

$$\frac{d\boldsymbol{\zeta}}{dt} = [\boldsymbol{\Omega}\boldsymbol{\zeta}]. \quad (29)$$

In some papers (see, for instance, [43, 44]) it was shown that the precession frequency $\boldsymbol{\Omega}$ consists of two fundamentally different parts

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_L + \boldsymbol{\Omega}_{\text{Th}},$$

where

$$\boldsymbol{\Omega}_L = -\frac{eg}{2m_0c} \left(\frac{1}{\gamma} \mathbf{H} - [\boldsymbol{\beta}\mathbf{E}] - \frac{\gamma}{\gamma+1}[\boldsymbol{\beta}[\boldsymbol{\beta}\mathbf{H}]] \right) \quad (30)$$

is the Larmor frequency and

$$\boldsymbol{\Omega}_{\text{Th}} = -\frac{e}{m_0c} \frac{\gamma}{\gamma+1} ([\boldsymbol{\beta}\mathbf{E}] + [\boldsymbol{\beta}[\boldsymbol{\beta}\mathbf{H}]]) \quad (31)$$

is the Thomas precession frequency which, as known, has the kinematical origin

$$\boldsymbol{\Omega}_{\text{Th}} = -\frac{1}{c} \frac{\gamma^2}{\gamma+1} [\boldsymbol{\beta}\mathbf{a}],$$

where \mathbf{a} is the particle acceleration.

Transforming in equations (30) and (31) the electromagnetic fields to the rest frame of a particle we obtain the simple result

$$\boldsymbol{\Omega}_L = -\frac{eg}{2m_0c} \frac{1}{\gamma} \mathbf{H}_0, \quad \boldsymbol{\Omega}_{\text{Th}} = -\frac{e}{m_0c} \frac{1}{\gamma+1} [\boldsymbol{\beta}\mathbf{E}_0].$$

The Thomas precession [45] is considered in a large number papers (see [25, 43-48] et al.). Here we want to show how the separation of spin equations into dynamical and kinematical parts can be carried out via covariant form of spin equation (21). For simplicity, as above, we limit ourselves by the case of homogeneous fields (BMT approximation).

The equation (21) is considerably simplified via introduction of a certain effective field with physically distinguished kinematical part which meets the Thomas precession

$$H_{\text{eff}}^{\alpha\beta} = \vec{H}^{\alpha\beta} + \frac{2m_0}{egc} v^{[\alpha} w^{\beta]}, \quad (32)$$

where

$$w^\beta = \frac{e}{m_0c} H^{\beta\rho} v_\rho,$$

$\vec{H}^{\alpha\beta}$ is the space-like part of an electromagnetic field

$$\vec{H}^{\alpha\beta} = H^{\alpha\beta} + \frac{1}{c^2} v^{[\alpha} v_{\rho} H^{\rho\beta]}, \quad v_{\alpha} \vec{H}^{\alpha\beta} = 0.$$

Then the spin equation (21) takes the simple form

$$\frac{d\Pi^{\mu\nu}}{d\tau} = \frac{eg}{2m_0c} H_{\text{eff}}^{[\mu\rho} \Pi_{\rho}^{\nu]}.$$

The equation (28) for vector ζ is also considerably simplified

$$\frac{d\zeta}{d\tau} = \frac{eg}{2m_0c} \left[\zeta \left(\mathbf{H}_{\text{eff}} - \frac{\gamma}{\gamma+1} [\beta \mathbf{E}_{\text{eff}}] \right) \right].$$

Separating here the kinematical part of fields according to (32)

$$\begin{aligned} \mathbf{H}_{\text{eff}} &= \vec{\mathbf{H}} + \frac{2}{g} \gamma^2 [\beta (\mathbf{E} + [\beta \mathbf{H}])], \\ \mathbf{E}_{\text{eff}} &= \vec{\mathbf{E}} + \frac{2}{g} \gamma^2 (\mathbf{E} + [\beta \mathbf{H}] - \beta (\beta \mathbf{E})), \end{aligned}$$

where $\vec{\mathbf{H}}$ and $\vec{\mathbf{E}}$ are components of $\vec{H}^{\alpha\beta}$, we obtain equation (29) with separated frequencies.

Thus one can compare the kinematical field

$$H_{\text{Th}}^{\alpha\beta} = \frac{2m_0}{egc} v^{[\alpha} w^{\beta]}$$

with Thomas precession.

One can also introduce the kinematical field dual to $H_{\text{Th}}^{\alpha\beta}$

$$E_{\text{Th}}^{\alpha\beta} = \frac{1}{2} \varepsilon^{\alpha\beta\rho\sigma} H_{\rho\sigma}^{\text{Th}} = \frac{2m_0}{egc} \varepsilon^{\alpha\beta\rho\sigma} w_{\rho} v_{\sigma}.$$

It is interesting that $E_{\text{Th}}^{\alpha\beta}$ is space-like tensor:

$$v_{\alpha} E_{\text{Th}}^{\alpha\beta} = 0,$$

and

$$E_{\text{Th}}^{\alpha\beta} H_{\alpha\beta}^{\text{Th}} = 0.$$

Another invariant of kinematical field is proportional to the square of acceleration

$$H_{\text{Th}}^{\alpha\beta} H_{\alpha\beta}^{\text{Th}} = -\frac{8m_0^2}{e^2 g^2} w_{\mu} w^{\mu}.$$

We note also that

$$\frac{1}{2} H_{\mu\nu} \Pi^{\mu\nu} = \frac{1}{2} \vec{H}_{\mu\nu} \Pi^{\mu\nu} = \frac{1}{2} H_{\text{eff}}^{\mu\nu} \Pi_{\mu\nu} = (\zeta \mathbf{H}_0). \quad (33)$$

Hence it follows that the Thomas precession gives no addition to the effective mass (3).

Thus we satisfy ourselves that the Thomas precession is a purely kinematical spin-orbit effect.

5 Correspondence principle in the spin theory

In the quantum theory of the Dirac particles the spin description is given via the Poincare-invariant spin operators [33]. Thereat the operator equations of motion are obtained from the Heisenberg proper time equations with the Dirac covariant Hamiltonian [12]. To found a correspondence between the classical and quantum equations of motion it is necessary to separate out the terms responsible for quantum Zitterbewegung. We remind that this phenomenon unlike the classical Zitterbewegung (see above) is accounted for interference of charge conjugate states (see, for example, [49, 50] et al.).

The problem of separation (not exception!) of Zitterbewegung is successfully solved by means of quantum theory with definite parity of operators [51]. It is found that the general classical relations like the spin mass (3), momentum invariant (26) and spin invariants (33) have the corresponding quantum analogues with even operators (see [29] for m , see [52] for $P_\alpha P^\alpha$, and see [53] for $H_{\mu\nu}\Pi^{\mu\nu}$). For instance, instead of (26) we obtain

$$\overset{\vee}{P}_\alpha \overset{\vee}{P}^\alpha = - \overset{\vee}{m} c^2, \quad (34)$$

where

$$\overset{\vee}{m} = m_0 \left(1 - \frac{\mu}{2c^2} H_{\alpha\beta} \overset{\vee}{\Pi}^{\alpha\beta} \right),$$

$\overset{\vee}{P}^\alpha$ and $\overset{\vee}{\Pi}^{\alpha\beta}$ are the even operators which correspond to \hat{P}^α and $\sigma^{\alpha\beta}$. Operator $\overset{\vee}{P}^\alpha$ passes into the ordinary spin operator with external field at $\hbar \rightarrow 0$, spin operator $\overset{\vee}{\Pi}^{\alpha\beta}$ (Hilgevoord-Wouthuysen operator) does not contain the Plank constant. It is interesting that Eq. (34) is invariant with respect to replacement of the degree of operators parity, i.e. $\vee \rightarrow \vee\vee \rightarrow \vee\vee\vee \rightarrow \dots$ (see [52]).

Operator equations of motion for $\overset{\vee}{P}^\alpha$ and $\overset{\vee}{\Pi}^{\alpha\beta}$ are the Heisenberg proper time equations with the Dirac or Dirac-Pauli squareable Hamiltonian taking into account the anomalous magnetic moment of an electron. The obtained in this way operator equations coincide completely in form with corresponding classical equations at $\hbar \rightarrow 0$, but provided the terms with the Plank constant in the latter equations should be also tended to zero (see also quasiclassical BMT approximation [12] et al.). At the same time this result means that the problem of correlation between the classical and quantum Zitterbewegung remains vague. It is possible that this problem will be solved via construction of a quantum theory with the invariant parity degree of operators, or other methods.

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